Available online at www.sciencedirect.com
ScienceDirect
International Journal of Multiphase Flow 32 (2006) 1287–1293
International Journal of Multiphase Flow
www.elsevier.com/locate/ijmulflow

Brief communication

Improved closure models for gas entrainment and interfacial shear for slug flow modelling in horizontal pipes

R.I. Issa *, M. Bonizzi, S. Barbeau

Department of Mechanical Engineering, Imperial College, Exhibition Road, London SW7 2AZ, UK

Received 28 November 2005; received in revised form 7 June 2006

Keywords: Slug flow; Interfacial friction; Gas entrainment; Two-fluid model

1. Introduction

In a paper by Issa and Kempf (2003), it was demonstrated that the two-fluid model in its one-dimensional, transient form can automatically capture the generation and subsequent motion of slugs in two-phase flow in horizontal and nearly horizontal pipes. In that work, the influence of closure models to represent frictional effects (both wall and interfacial), on the prediction of slug initiation and characteristics, once generated, was discussed. It was found that the correlation of Taitel and Dukler (1976) for the friction factor for interfacial shear gave good results for the range of gas velocities considered. However, the correlation is appropriate for smooth interfaces, while it is well known that at high gas velocities, the gas/liquid interface becomes wavy (rough), thereby enhancing frictional effects between the liquid and gas. Andritsos and Hanratty (1987) had developed an alternative correlation for situations when the interface is wavy; it is this correlation that has been implemented in the slug capturing numerical procedure of Issa and Kempf and the results are presented in this paper. It will be shown that as the gas velocity increases, the Andritsos and Hanratty correlation begins to exert greater influence on the quality of the prediction of slug characteristics, giving better agreement with experiment.

In slug flow, gas is often entrained from the large elongated gas (often called Taylor) bubble into the body of the liquid slug (also referred to as “aeration”), and this is believed to have a significant effect on the slug behaviour. The phenomenon is a direct consequence of the strong turbulent action induced by the large vortex motion set up at the slug front by the entry of the higher velocity liquid from the preceding liquid film. This leads to the fragmentation of the tail of the elongated Taylor bubble into small gas pockets that are entrapped in the flowing liquid. It is hence desirable to model this phenomenon in order to improve the accuracy of the prediction of slug characteristics. In the development of the slug capturing methodology, Bonizzi and Issa (2003) extended the capability to account for the entrainment of small gas bubbles in the body of the liquid

* Corresponding author. Tel.: +44 20 75947038; fax: +44 20 75941472.
E-mail address: r.issa@imperial.ac.uk (R.I. Issa).

0301-9322/$ - see front matter
doi:10.1016/j.ijmultiphaseflow.2006.06.003

slug. In that work, various correlations for the rate of entrainment of gas were tested and the one that was found to give the best results was due to the hydraulic jump analogy. However, that analogy considers only hydrodynamic effects and does not account for fluid properties such as surface tension and viscosity which are bound to play a major role in determining the entrainment process (for example, it is known that entrainment rate in a gas/oil system is different from that in a water/gas system). Recently, Brauner and Ullamn (2004) proposed a new correlation for the entrainment rate which accounts for the physics of the process and features the fluid properties. In the present paper, this correlation is adapted and the resulting slug flow predictions are tested against measurements.

2. Two-fluid model

The basis of the two-fluid model is the formulation of two sets of conservation equations for the balance of mass, momentum and energy for each of the phases. The one-dimensional form of the model is obtained by integrating (area averaging) the flow properties over the cross-sectional area of the flow. The transfer of momentum and energy between the walls and the fluids and between the fluids themselves is accounted for via source terms in the equations that are based on empirical correlations. The present study concerns isothermal flow so the equations solved are those for the conservation of mass and momentum only. For one-dimensional stratified and slug flow they are

$$\frac{\partial(\rho_G\alpha_G)}{\partial t}+\frac{\partial(\rho_G u_G\alpha_G)}{\partial x}=-m_b^\bullet \tag{1}$$

$$\frac{\partial(\rho_L\alpha_L)}{\partial t}+\frac{\partial(\rho_L u_L\alpha_L)}{\partial x}=-m_b^\bullet \tag{2}$$

$$\frac{\partial(\rho_G\alpha_G u_G)}{\partial t}+\frac{\partial(\rho_G\alpha_G u_G^2)}{\partial x}=-\alpha_G\frac{\partial p}{\partial x}+\rho_G\alpha_G g\sin\beta+F_{wG}+F_i \tag{3}$$

$$\frac{\partial(\rho_L\alpha_L u_L)}{\partial t}+\frac{\partial(\rho_L\alpha_L u_L^2)}{\partial x}=-\alpha_L\frac{\partial p}{\partial x}-\rho_L\alpha_L g\frac{\partial h}{\partial x}\cos\beta+\rho_L\alpha_L g\sin\beta+F_{wL}-F_i \tag{4}$$

These equations are complemented by the compatibility relation:

$$\alpha_G+\alpha_L=1 \tag{5}$$

In the above equations, subscripts G, L, and i refer to the gas and liquid phases, and the interface, respectively. The axial coordinate is x, the density is ρ, the phase fraction is α, the velocity is u, the mass transfer per unit volume between the phases is $m_b^\bullet$, the interface (and gas) pressure is p, the pipe inclination is β, the height of the liquid surface (assumed flat) above the pipe bottom is h and the acceleration due to gravity is g. The liquid is assumed to be incompressible, while the gas is taken to be compressible obeying the ideal-gas equation of state. The terms F stand for the frictional forces per unit volume between each phase and the wall, and between the phases at their interface. These shear forces are determined from commonly used friction factor correlations (for details see Issa and Kempf, 2003). Of interest here is the interfacial friction which is given by

$$F_i=-\frac{1}{2}f_i\rho_G|u_G-u_L|(u_G-u_L)S_i/A \tag{6}$$

where, A is the pipe cross-sectional area and S_i is the interfacial width. The friction factor at the interface, f_i is the subject of investigation here.

3. Interfacial friction at wavy interfaces

In previous slug capturing calculations carried out at low gas velocities (Issa and Kempf, 2003), the interfacial friction factor f_i was assumed to be the same as that for gas-wall friction, f_g. Both were based on the correlation of Taitel and Dukler (1976) which gives $f_i=C_iRe_i^{-n_i}$, where Re_i is defined as $Re_i=\frac{D_G(u_G-u_L)}{\nu_G}$ and D_G is the hydraulic diameter of the gas phase. The coefficients C_i and n_i respectively take the values of 0.046 and 0.25 if the flow is turbulent ($Re_i>2100$), or 16 and 1 if the flow is laminar ($Re_i\leqslant 2100$).

The assumption of equal gas and interface friction factors in stratified flow is only valid when the gas–liquid interface is smooth. At high gas velocities ($u_G>5$ m/s), the interface is known to become wavy, resulting in much increased interfacial shear. The interface then acts as a rough surface over which the gas, being the faster fluid, flows. These effects cannot in general be captured by a one-dimensional model as they arise from multi-dimensional phenomena. This therefore calls for a correlation appropriate to wavy interfaces and here the correlation proposed by Andritsos and Hanratty (1987) is implemented and evaluated.

Andritsos and Hanratty observed that the ratio f_i/f_G, remains roughly equal to unity until a certain value of the superficial gas velocity U_{Gt} corresponding to the transition from a smooth interface to a regime of large irregular waves is reached. That velocity was found to be $U_{G,t} = 5\left(\frac{\rho_{Go}}{\rho_G}\right)^{1/2}$ where ρ_{G0} and ρ_G are respectively the gas density at atmospheric and operating pressures. They then proposed the following relation for the determination of f_i which is expressed in term of the ratio f_i/f_G. Hence

$$f_i/f_G = 1 + 15\sqrt{\frac{h}{D}}\left[\frac{U_G}{5}\sqrt{\frac{\rho_G}{\rho_{Go}}} - 1\right] \quad \text{for } U_G \geqslant U_{G,t} \tag{7}$$

otherwise, $f_i/f_G = 1$ for $U_G < U_{G,t}$.

4. Gas entrainment

In order to account for the process of aeration within the framework of the slug capturing methodology, Bonizzi and Issa (2003) introduced a model based on the solution of an additional transport equation for the fraction of gas bubbles that are entrained into the slug, α_B. The new equation takes the form

$$\frac{\partial(\rho_G \alpha_B)}{\partial t} + \frac{\partial(\rho_G u_B \alpha_B)}{\partial x} = m_b^\bullet \tag{8}$$

where $m_b^\bullet$ is the local rate of gas bubbles entrained (at the slug front) or rejected (at the slug tail) per unit volume. The velocity of the small bubbles u_B is determined from an equation of motion for an individual bubble in which equilibrium between the local pressure gradient (obtained from computed pressure field) and the drag force on individual bubbles is assumed. Another necessary modification is to treat the liquid phase as a mixture composed of the liquid and dispersed small bubbles; this entails the replacement of the liquid density in the continuity and momentum Eqs. (2) and (4) by that of the mixture, thus

$$\rho_L \Rightarrow \alpha_L \rho_L + \alpha_G \rho_G$$

The entrainment rate $m_b^\bullet$ plays a crucial role in determining the amount of gas entrained in the body of the slug and must be specified by a closure relation. Bonizzi and Issa (2003) used a few relationships available in the literature for the specification of $m_b^\bullet$ and found that the most accurate one was that based on the analogy to hydraulic jumps as proposed by Jepson (1987). The argument for this being that since the entrainment is a consequence of the liquid pick-up process from the liquid film into the slug, the phenomenon is similar to the air carry-under and bubble dispersion process associated with plunging jets occurring in hydraulic jumps. Among the many available correlations for the entrainment rate in hydraulic-jumps, the one recommended by Chanson (1996) was used since it is reportedly to be the most widely used in engineering practice. It is expressed as

$$M_b^\bullet = \rho_G A_{lf}(u_t - u_{Lf})\varsigma(Fr - 1)^\varepsilon \tag{9}$$

where $M_b^\bullet$ is the entrained mass flux of gas bubbles from which $m_b^\bullet$ in Eq. (8) can be determined, and u_t and u_{LF} are the velocities of the slug body and the liquid film ahead of it, respectively. Coefficients ζ and ε depend on the Froude number and take respectively the values of 0.018 and 1.245 for $2.5 < Fr < 7$ and 0.014 and 1.4 for $7 < Fr < 30$. The Froude number Fr is defined by $Fr = \frac{u_t - u_{Lf}}{\sqrt{gA_{Lf}/S_i}}$, where A_{Lf} represents the area occupied by the liquid film.

Although Bonizzi and Issa obtained very good agreement with measurements using this correlation for the entrainment rate, their comparisons were limited to air/water systems. It is evident from the Eq. (9) that the expression is independent of fluid properties, in particular, viscosity and surface tension. This is at variance with experimental observation; hence a correlation that involves fluid properties should be employed. To this end, the recently published correlation of Brauner and Ullamn (2004) is introduced. That correlation was derived on the basis of the balance between the turbulence kinetic energy flux, associated with the film jet, and the surface energy production. This leads to an entrainment rate (mass flux) given by

$$M_{\mathrm{b}}^{\bullet} = \frac{A\rho_{\mathrm{G}}}{400C_j} d_{\mathrm{crit}}(We - We_{\mathrm{cr}})\alpha_{\mathrm{Lf}}(u_{\mathrm{t}} - u_{\mathrm{Lf}}) \tag{10}$$

where C_j is a coefficient of the order of unity (taken here to be 1) and the critical bubble diameter d_{crit} is given by $d_{\mathrm{crit}} = \sqrt{\left[\frac{0.4\sigma_{\mathrm{GL}}}{|\rho_{\mathrm{L}}-\rho_{\mathrm{G}}|g}\right]}$. The Weber numbers We and We_{cr} are defined by $We = \frac{\rho_{\mathrm{L}}(u_{\mathrm{L}}-u_{\mathrm{Lf}})^2 D}{\sigma_{\mathrm{GL}}}$ and $We_{\mathrm{cr}} = \frac{100C'}{d_{\mathrm{crit}}/D}$ where C' takes the value of 0.5 and D is the pipe diameter.

5. Results

5.1. Interfacial shear

The correlation of Andritsos and Hanratty was implemented in an existing computational code (described by Bonizzi and Issa, 2003) that has the capability of capturing slug flow using the two-fluid model. The results of the calculations for air water systems (in the absence of slug aeration) are compared with both experimental data and the smooth interface correlation of Taitel and Dukler. The reason for not accounting for the effects of gas entrainment was to enable the study the influence of the interfacial shear in isolation. The measurements were obtained by Manolis (1995) from the WASP facility at the Chemical Engineering Department of Imperial College. Simulations were carried out for different values of superficial gas and liquid inlet velocities as shown in Table 1. The length of pipe considered was 36 m and the number of computational cells was 1250 which corresponds to grid spacing of about 0.4 pipe diameter. This size of mesh was found (through successive mesh refinement) to be appropriate to yield gird-independent solutions. The pressure at outlet was taken as atmospheric.

The statistically averaged slug frequencies obtained for each case using the two correlations for the computation of the interfacial friction factor are presented in Fig. 1 together with experimental data. At lower superficial gas velocity ($U_{\mathrm{G}} < 5$ m/s), both correlations provide excellent agreement for slug frequencies. The agreement between the two correlations at these velocities is not surprising since the friction factors should be equal for a smooth interface. On the other hand, for superficial gas velocity higher than 5 m/s, the results from the two correlations begin to differ, with the Andritsos and Hanratty giving consistently

Table 1
Cases studied for interfacial shear validations

Runs	U_{G} (m/s)	U_{L} (m/s)
Case 1	10.474	0.761
Case 2	6.837	0.772
Case 3	8.344	0.49
Case 4	3.135	0.534

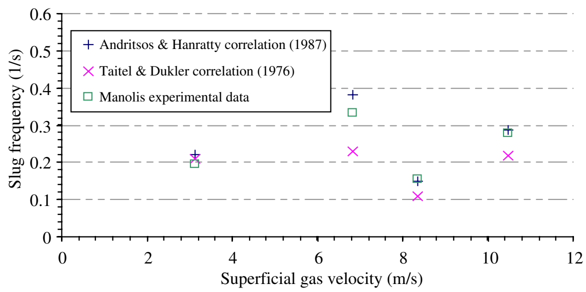

Fig. 1. Predicted and measured slug frequency.

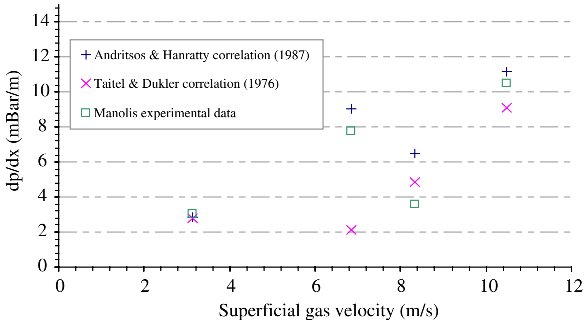

Fig. 2. Predicted and measured pressure gradient.

higher values that are in closer agreement with the data. This is to be expected since the propensity of the flow to slug is enhanced by higher frictional forces (as was shown by Issa and Kempf (2003)).

Fig. 2 presents the results for the pressure gradient along the pipe. At lower gas velocities, the pressure gradient is predicted well with both correlations. At higher gas velocities the Andritsos and Hanratty correlation once again tends to predict the data better, with the exception of case 3. Had it not been for this one computation, more definitive conclusions could have been drawn regarding the efficacy of the Andritsos and Hanratty correlation. Nonetheless, it can be said that it has generally led to better predictions in most cases.

It should be pointed out that the Andritsos and Hanratty correlation was only tried in slug flow at atmospheric pressure. It has been suggested by some workers in the field that the correlation may not be valid in stratified flow at high pressures (wherein the interface may be smooth even at high gas velocities). Indeed, subsequent computations in the stratified flow regime, carried out by the present authors, have confirmed this limitation.

5.2. Gas entrainment

The correlation of Brauner and Ullman for the entrainment rate was also implemented and tested against the hydraulic jump correlation. The computations were validated against two sets of experimental data. The first set is that of Felizola and Shoham (1995) who used air and kerosene in their experiment; the density, viscosity and surface tension of the kerosene were respectively 800 kg/m^3, 0.0016 kg/(m s) and 0.028 N/m. The pipe used was 15 m long and 0.051 m in internal diameter. The second set of experiments was that due to Gregory et al. (1978) who studied air and light oil slug flow in a 17 m pipe of 0.0512 m internal diameter

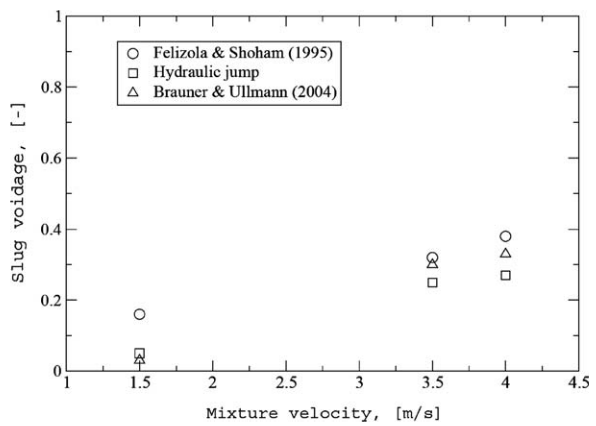

Fig. 3. Predicted and measured gas fraction; data of Felizola and Shoham.

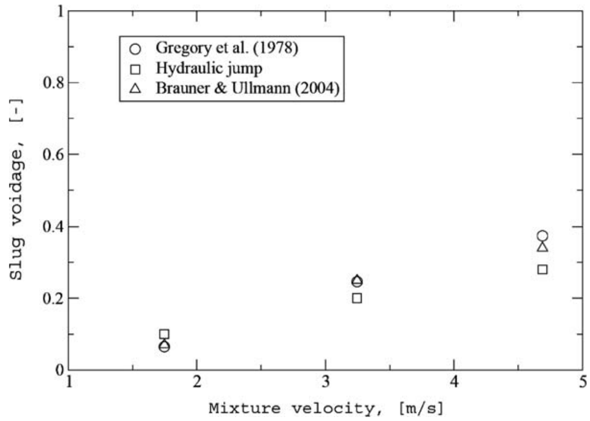

Fig. 4. Predicted and measured gas fraction; data of Gregory et al.

at a pressure of 2.55 bar. The density and viscosity of the light oil were 858 kg/m^3 and 0.00675 kg/(m s) respectively. Simulations were performed for liquid superficial velocities of 0.5 m/s for the Felizola and Shoham experiment and 0.305 m/s for the Gregory et al. data. The computations were carried out with a mesh spacing of $0.5D$ (which once again was established to give grid-independent solutions); at inlet, steady state conditions were assumed.

Figs. 3 and 4 show the computed slug void fraction for different mixture velocities compared against the above mentioned sets of data. It is evident that in both cases the agreement between the theory and experiment is substantially improved at moderate and high mixture velocities by using the Brauner and Ullman correlation. The discrepancies between calculations and data are typically reduced from around 25% to only 10%, which is a significant reduction. This improvement is undoubtedly brought about by the accounting for fluid properties in the new correlation, unlike the hydraulic jump which was developed for air/water systems and does not feature fluid properties.

6. Conclusions

In order to improve the slug capturing capability of the one-dimensional, transient two-fluid model, accurate closure relations are needed. Here, two improved closure models have been implemented and tested against available experimental measurements. The first relates to the friction factor for wavy interfaces in stratified flow. The correlation of Andritsos and Hanratty for rough interfaces was introduced and validated in slug flow at atmospheric pressures and was mostly found to improve the predictions over those from the smooth interface correlation of Taitel and Dukler. Further work on such correlations is needed for systems at high pressures and higher gas velocities.

The second development relates to the introduction of the recently published correlation for the rate of gas entrainment into liquid slugs by Brauner and Ullman. The new correlation was found to give improved predictions at moderate and high mixture velocities over the previously used hydraulic jump analogy. It is believed that the superior performance of the new correlation is due to the fact that, unlike the hydraulic jump, it involves the fluid properties.

Acknowledgements

The Authors wish to acknowledge the financial support given by the Engineering and Physical Sciences Research Council (EPSRC) and to the following industrial organisations: Aspen Technology; BG International; BP Exploration; Chevron; ConocoPhillips; Enterprise Oil; Granherne; Institutt for Energiteknikk; Institut Français du Pétrole; Marathon Oil; Mobil North Sea; Norsk Hydro; Scandpower; TotalFinaElf.

References

Andritsos, N., Hanratty, T.J., 1987. Influence of interfacial waves in stratified gas–liquid flow. AIChE J. 33, 444–454.

Bonizzi, M., Issa, R.I., 2003. A model for simulating gas bubble entrainment in two-phase horizontal slug flow. Int. J. Multiphase Flow 29, 1685–1717.

Brauner, N., Ullamn, A., 2004. Modelling of gas entrainment from Taylor bubbles. Part A: Slug flow. Int. J. Multiphase Flow 30, 239–272.

Chanson, H., 1996. Air Bubble Entrainment in Free-Surface Turbulent Shear Flow. Academic Press.

Felizola, H., Shoham, O., 1995. A unified model for slug flow in upward inclined pipes. J. Energy Res. Technol. 117, 7–11.

Gregory, G.A., Nicholson, M.K., Aziz, K., 1978. Correlation of the liquid volume fraction in the slug for horizontal gas–liquid slug flow. Int. J. Multiphase Flow 4, 33–39.

Issa, R.I., Kempf, M.H.W., 2003. Simulation of slug flow in horizontal and nearly horizontal pipes with the two-fluid model. Int. J. Multiphase Flow 29, 69–95.

Jepson, W.P., 1987. The flow characteristics in horizontal slug flow. In: Proceedings of 3rd International Conference on Multiphase Flow, The Hague, Netherlands, pp. 187–198.

Manolis, I.G., 1995. High pressure gas–liquid slug flow. Ph.D. Thesis, Chem. Eng. Dept., Imperial College, London, UK.

Taitel, Y., Dukler, A.E., 1976. A model for predicting flow regime transitions in horizontal and near horizontal gas–liquid flow. AIChE J. 22, 47–55.